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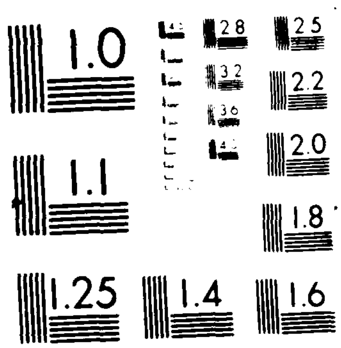
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Short Paper

FEEDBACK STABILIZATION OF DISTRIBUTED SYSTEMS

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Introduction. We consider stabilization (minimum time to the zero solution) of wave and beam equations via endpoint velocity control. In general, consider a control system described by a partial differential equation, with dependent variable denoted $w = w(t,x)$, which admits $V = \frac{\partial}{\partial t}$ as a symmetry generator and hence a conservation law which implies conservation of energy, denoted $e(w(t,\cdot))$. For the problems considered, it is shown that $\frac{d}{dt} e(w(t,\cdot))$ is a convex function of the control u; the value u^* which minimizes this can be expressed in feedback form, i.e., if $w(t,\cdot)$ belongs to a Banach space B and $l: B \to \mathbb{R}^k$ is a closed linear map (a finite dimensional measurement on the state of the system; the output of sensors), then the optimal control will have the form $u^* = l(w(t,\cdot))$.

The energy decay of solutions of wave, or beam, equations via boundary control has received much attention in the recent literature. In [1], the wave equation

$$\rho(x) \frac{\partial^2 w}{\partial t^2} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} (\alpha_{ij}(x)) \frac{\partial w}{\partial x_j} = 0 , \tag{1}$$

with $t \geq 0$, $x \in \Omega \subset \mathbb{R}^n$, Ω open, bounded, connected and with boundary $\partial\Omega = \Gamma$, is studied. Assume $\rho(x) \geq \rho_0 > 0$; the matrix $A = (\alpha_{ij}(x))$ is positive definite symmetric (i.e., the inner product $(\xi, A(x)\xi) \geq \alpha_0 |\xi|^2$ for some $\alpha_0 > 0$). Partition the boundary as $\Gamma = \Gamma_1 \cup \Gamma_2$ with

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Γ_1 relatively open and require $w(t,x) = 0$ on Γ_1, $w(t,x) = h(t,x)$ a control for $x \in \Gamma_2$. The energy of a solution w of (1) is

$$e(w(t,\cdot)) = (\frac{1}{2}) \int_\Omega [\rho(x) w_t^2(t,x) + (w_x(t,x), A(x) w_x(t,x))] dx . \tag{2}$$

If w satisfies (1) and $\nu(x)$ denotes an outward normal to $\partial\Omega$, the divergence theorem yields

$$\frac{d}{dt} e(w(t,\cdot)) = \int_\Gamma w_t(t,x)(w_x(t,x), A(x)\nu(x)) dx . \tag{3}$$

Change control from "position h" to velocity $u(t,x) = h_t(t,x) = w_t(t,x)$, $x \in \Gamma_1$. From (3), a natural choice to decrease energy is

$$u(t,x) = w_t(t,x) = -(w_x(t,x), A(x)\nu(x)), \quad x \in \Gamma_1 . \tag{4}$$

A great deal of the literature is devoted to an analysis of (3) with control choice (4). In [1] an invariance principle (extended LaSalle principle as given by Hale) is used to show $e(w(t,\cdot)) \to 0$ as $t \to \infty$. Under more special conditions on the shape of Ω and with $\rho(x) = 1$, $A(x) = I$, the convergence rate $e(w(t,\cdot)) = \frac{c}{(1+t)},$ $t \geq 0$, is obtained, where c depends on the initial conditions. It is well known [2], [3, Thm. 1], that solutions of the one-dimensional wave equation can be driven to zero in finite time. conditions which insure this for quasi-linear hyperbolic equations can be found in [4]. For more general systems, exponential asymptotic estimates of rate of energy decay can be found in [5], [6]. Recent results involving the physical character of sensors and actuators in energy decay are given in [7]. Energy methods for boundary control of the beam equation can be found in [8], [9].

Our purpose, here, will be to stress geometry, optimality, the feedback nature of the controls (with it the sensor placement problem) and to suggest interesting areas for further research.

1. The Endpoint Control of a Vibrating String

Let $w(t,x)$ denote the vertical displacement at position x, $0 \leq x \leq 1$, and time t of a string of length 1. The equation of motion is taken as

$$w_{tt} = w_{xx}, \quad 0 \leq x \leq 1, \ t \geq 0 . \tag{5}$$

Initial Conditions (IC): $w(0,x) = \phi(x)$, $w_t(0,x) = \eta(x)$ (6a)

Boundary Conditions(BC): $w(t,0) = 0$, $w_t(t,1) = u(t) = \text{control}$ (6b)

The problem is to use the velocity control (or one could use height or right endpoint position control) to drive the solution to $w(t_1,x) = w_t(t_1,x) = 0$, $0 \leq x \leq 1$, for minimum time t_1. It is well known, [2], [3], that for the length one string and arbitrary initial data this can be achieved in time $t_1 = 2$ and no smaller time suffices. Indeed, calculations give the explicit formula

$$u^*(t) = \begin{cases} -(\frac{1}{2})(\phi'(1-t) - \eta(1-t)), & 0 \leq t \leq 1 \\ -(\frac{1}{2})(\phi'(t-1) + \eta(t-1)), & 1 < t \leq 2 \end{cases} \tag{7}$$

For this simple problem, we see in (2) that $\Gamma_1 = \{0\}$, $\Gamma_2 = \{1\}$, while if we denote the solution of (5), (6) as $w(t,x,u)$ to explicitly show the u dependence,

$$\frac{d}{dt} e(w(t,\cdot,u)) = u(t)w_x(t,1,u) . \tag{8}$$

A natural choice (as in (4)) is to choose $u(t) = -kw_x(t,1,u)$, $k > 0$. The interesting thing (as will be shown) is that the right side of (8) is quadratic in u and has minimum at $k = 1$, i.e., $u^*(t) = -w_x(t,1)$ is the optimal control. Indeed, one can see this by considering (5) with initial conditions (6a) and (nonstandard) boundary conditions $w(t,0) = 0$, $w_t(t,1) = -w_x(t,1)$. This problem can be explicitly solved by standard techniques; one readily finds that the solution satisfies $w_t(t,1) = u^*(t)$ as given by (7).

Finally, for the endpoint controlled string, we have

Proposition 1. The control choice $u^*(t)$ which pointwise minimizes $\frac{d}{dt}e(w(t,\cdot,u)) = u(t)w_x(t,1,u)$ satisfies $u^*(t) = -w_x(t,1,u^*)$ and is the optimal control.

To prove this, solve (5) with initial conditions (6a) and boundary conditions (6b), leaving $u(t)$ arbitrary, in a region R containing the right boundary $x = 1$. Explicitly, for $R = \{(t,x)\colon\ x + t > 1,\ x - t > 0,\ x \leq 1\}$ one may show

$$w(t,x,u) = -(\frac{1}{2})\phi(2-x-t) + (\frac{1}{2})\phi(x-t) + (\frac{1}{2})\int_{x-t}^{2-x-t} \eta(s)ds + \phi(1) + \int_0^{x+t-1} u(s)ds\,.$$

This allows us to compute $\lim_{x\to 1^-} w_x(t,x,u)$, which, for $0 \leq t \leq 1$, is $w_x(t,1,u) = \phi'(1-t) - \eta(1-t) + u(t)$. Substituting this in the equation for the time derivative of $e(w(t,\cdot,u))$ shows $\frac{d}{dt}e(w(t,\cdot,u)) = u(t)[\phi'(1-t) - \eta(1-t) + u(t)]$, $1 \leq t \leq 1$. Let $f(u) = u(\phi'-\eta+u)$. This quadratic has a minimum at $u^*(t) = -(\frac{1}{2})(\phi'(1-t) - \eta(1-t))$, $0 \leq t \leq 1$. A similar analysis shows that for $1 < t \leq 2$, $u^*(t) = -(\frac{1}{2})(\phi'(t-1) + \eta(t-1))$. Comparison with (7) shows u^* is optimal.

The point to be stressed is the quadratic nature of the right side eq. (8) which assures an absolute, pointwise, minimum. It is also, geometrically, interesting to note that the intuitive idea of choosing the right endpoint (i.e., $w_t(t,1)$) to be the negative of the slope of the wave at this endpoint (i.e., $-w_x(t,1)$), is optimal.

2. Endpoint Energy Control of a Beam

For ease of exposition we use the overly simple beam model

$$\rho w_{tt} + EIw_{xxxx} = 0, \quad 0 \le x \le l, \ t \ge 0 \tag{9}$$

I.C.: $w(0,x) = \phi(x), \quad w_t(0,x) = \eta(x), \ 0 \le x \le 1$

We assume a free right end yielding right boundary conditions

$$\text{B.C.(R):} \ w_{xx}(t,l) = w_{xxx}(t,l) = 0$$

The left boundary conditions will involve the controls and yield three natural problems. Before discussing these, we note the energy now is

$$e(w(t,\cdot)) = (\frac{1}{2}) \int_0^l (\rho w_t^2 + EIw_{xx}^2)dx \tag{10}$$

which, using B.C.(R), satisfies

$$\frac{d}{dt} e(w(t,\cdot)) = EIw_t(t,0)w_{xxx}(t,0) - EIw_{xt}(t,0)w_{xx}(t,0) . \tag{11}$$

Problem 1. The left endpoint velocity, i.e. $w_t(t,0)$, is the control and this end of the beam is held so as to always have $w_x(t,0) = 0$. This gives left boundary conditions

B.C.(L): $w_t(t,0) = u(t) = \text{control}, \quad w_{xt}(t,0) = 0$

Problem 2. Consider the left end of the beam fixed at zero with the initial slope, $w_x(t,0)$, the control. More precisely, we take the time rate of change of the initial slope as control.

B.C.(L): $w(t,0) = 0$ (or $w_t(t,0) = 0$), $\quad w_{xt}(t,0) = u(t) = \text{control}$

Problem 3. Both initial (left end) height and slope are controllable.

B.C.(L): $w_t(t,0) = u_1(t), \quad w_{xt}(t,0) = u_2(t).$

Motivated by the results of the one-dimensional wave equation, we see each problem suggests a natural control choice to decrease energy, i.e., in problem 1 choose $u(t) = -w_{xxx}(t,0)$; in problem 2 choose $u(t) = w_{xx}(t,0)$, in problem 3 choose $u_1(t) = -w_{xxx}(t,0)$, $u_2(t) = w_{xx}(t,0)$. Are these choices (or positive multiples of them) optimal? For the sake of brevity, we examine only problem 2 for which (11) becomes

$$\frac{d}{dt} e(w(t,\cdot,u)) = -EI w_{xt}(t,0) w_{xx}(t,0,u) \tag{12}$$

$$= -EI u(t) w_{xx}(t,0,u)$$

where (again) stress of dependence of w on u is made clear where needed. The choice $u(t) = w_{xx}(t,0)$ physically means pick the velocity of the initial slope to be the initial curvature; a reasonable scheme! Also, physically, one expects $|u|$ sufficiently large implies $\frac{d}{dt} e(w(t,\cdot,u)) > 0$ while $u = 0$ implies $\frac{d}{dt} e(w(t,\cdot,u)) = 0$, i.e., $\frac{d}{dt} e(w(t,\cdot,u))$ can be expected to be convex in u with (when $w_{xx}(t,0,u) \neq 0$) a negative absolute minimum. To compute, explicitly, one must calculate $w_{xx}(t,0,u)$. This may be accomplished via the Heaviside operational calculus, see [10, pg. 51]. Briefly, let $p = \frac{\partial}{\partial t}$, $h(t) = w_x(t,0)$ so $ph = u$, w^H denote the solution of (9) with homogeneous boundary conditions and (symbolically) express the solution of problem 2 as $w(t,x,u) = w^H(t,x) + T(x)h(t)$. The nonhomogeneous part, i.e. $v = Th$, must satisfy $(EIT_{xxxx} + \rho p^2 T)v = 0$, $T_x(0) = 1$, $T(0) = 0$, $T_{xx}(l) = 0$, $T_{xxx}(l) = 0$. Considering p as a parameter, and letting $\alpha^2 = \rho p^2/EI$, one finds

$$T(x) = c_1 e^{x\sqrt{\alpha/2}} \cos(x\sqrt{\alpha/2}) + c_2 e^{x\sqrt{\alpha/2}} \sin(x\sqrt{\alpha/2}) + c_3 e^{-x\sqrt{\alpha/2}} \cos(x\sqrt{\alpha/2}) + c_4 e^{-x\sqrt{\alpha/2}} \sin(x\sqrt{\alpha/2})$$

We are interested in $T_{xx}(0)h$, from which it follows only c_2, c_4 are of importance. Using the boundary data and T as above, with tedious calculations one finds $c_2 \sim \sqrt{p}$, $c_4 \sim \sqrt{p}$. Noting that α is a constant multiple of p, $T_{xx}(0)h \sim p^{3/2}h = p^{1/2}u$ which (see [10]) implies

$$T_{xx}(0)h(t) \sim \frac{d}{dt} \int_0^t \frac{u(s)}{\sqrt{t-s}}\, ds .$$

Finally, for a computable constant c, we have the behavior

$$\frac{d}{dt}\, e(w(t,\cdot,u)) \sim u(t) w^{H}_{xx}(t,0) + cu(t) \frac{d}{dt} \int_0^t \frac{u(s)}{\sqrt{t-s}}\, ds\,. \tag{13}$$

Questions: 1. Can one carry out the above computations precisely and by choosing u to minimize the right side of (13), explicitly calculate an optimal (for minimizing time to zero of $e(w(t,\cdot,u))$ control u^*?

2. Can one use $e(w(t,\cdot)) = y_1(t)$, $w_{xx}(t,0) = y_2(t)$ as two of a finite number (say n) of measurements on a solution w such that $y_1, \cdots, y_n$ satisfy a controlled system of ordinary differential equations? If so, one would use control to drive y_1 to zero.

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